[5] P. D. Coleman, "State of the art: Background and recent developments—Millimeter and submillimeter waves," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-11, pp. 271–288, Sept 1963.
[6] P. A. Cerenkov, *Compt. Rend. Acad. Sci. USSR*, vol. 2, pp. 451–454, 1934.
[7] I. Frank and I. Tamm, "Coherent visible radiation of fast electrons passing through matter," *Compt. Rend. Acad. Sci. USSR*, vol. 14, pp. 109–114, 1937.
[8] V. L. Ginzburg and I. M. Frank, "Radiation from a uniformly moving electron passing from one medium to another," Zh Eksp. Teor. Fiz., vol. 16, p. 15, 1946.
[9] G. Beck, "Contribution to the theory of the Cerenkov effect," *Phys. Rev.*, vol. 74, no. 7, pp. 795–803, Oct. 1948.
[10] V. E. Pafomov, "Radiation from an electron traversing a slab," *Sov. Phys.—JETP*, vol. 6, pp 829–830, 1958.
[11] G. M. Garibian, "Contribution to the theory of transition radiation," *Sov. Phys.—JETP*, vol. 6, pp. 1079–1085, June 1958.
[12] G. M. Garibian, "Radiation from a charged particle traversing a layered medium," *Sov. Phys.—JETP*, vol. 8, pp. 1003–1006, June 1959.
[13] A. G. Sitenko and V. S. Tkalich, "Cerenkov effect in the motion of a charge above a boundary between two media," *Sov. Phys.—Tech. Phys.*, vol 5, pp. 981–991, 1960.
[14] V. E. Pafomov, "Radiation interference effects in stratified media," *Sov. Phys.—Tech. Phys.*, vol. 8, no. 5, pp. 412–414, 1963.
[15] F. G. Bass and V. M. Yakovenko, "Theory of radiation from a charge passing through an electrically inhomogeneous medium," *Sov. Phys. Usp.*, vol. 8, no. 3, pp. 420–444, 1965.
[16] S R. Seshadri, "Transition radiation from a plasma boundary," *Can. J. Phys.*, vol. 50, pp. 2244–2252, 1972.
[17] S. R Sharma and K. C. Swami, "Transition radiation by a charged particle moving across a plasma sheet," *J. Appl Phys.*, vol. 47, pp. 74–77, Jan. 1976.
[18] K. S. H. Lee and C. H. Papas, "Antenna radiation in a moving dispersive medium," *IEEE Trans. Antenna Propagat.*, vol AP-13, pp. 799–804, May 1965.
[19] K. Hazama, T. Shiozawa, and I. Kawano, "Effect of a moving dielectric half-space on the radiation from a line source," *Radio Sci.*, vol. 4, pp. 483–488, May 1969.
[20] B. R. Chawla and H. Unz, *Electromagnetic Waves in Moving Magneto-Plasmas*. The University Press of Kansas, 1969.
[21] M. Kobayashi and R. Terakado, "New view on an anisotropic medium and its application to transformation from anisotropic to isotropic problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-27, pp. 769–775, Sept. 1979.

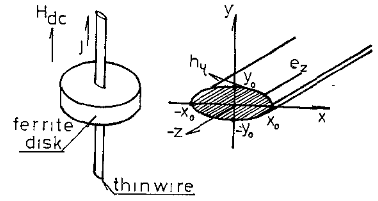
Radial Line Transducer

E. SAWADO

I. Introduction

The purpose of this paper is to give a new excitation method of a radial-electromagnetic wave by a metallic cylinder with elliptical cross section. It was ascertained [1] that the radial wave propagates in a medium of permeability of quantity $\mu_\perp$. The quantity $\mu_\perp$ is given by $\mu_\perp = (\mu^2 - \kappa^2)/\mu$, where μ and κ are the diagonal and nondiagonal components of the tensor permeability of a gyrotropic medium, respectively. The radial wave has interesting properties that this mode has not cutoff below the critical frequency $\omega = \gamma(BH)^{1/2}$, where ω is the angular frequency, $\gamma = 1.76 \times 10^7$ ((oe s)$^{-1}$ in CGS unit), $B = \mu_0(H + M_s)$, the magnetic flux density, H the magnetic field, and M_s the saturation magnetization. Ganguly and Webb [2] presented an initial theory and some experiments for a magnetostatic surface wave single bar transducer. These investigations have concluded that the lowest operating frequency of Ganguly-type delay line is $\gamma(BH)^{1/2}$. Below this cutoff, no surface modes can exist. In view of the above, investigation of a radial wave type delay line should produce developments in low frequency microwave (0.5 to 1.5 GHz) applications.

Manuscript received February 1, 1982; revised May 17, 1982.
The author is with the Faculty of Technology, Tokyo Metropolitan University, 1-1 Fukazawa 2-Chome, Setagaya-ku, Tokyo, Japan.



(a) (b)

Fig. 1. Schematics for radial-line transducer with dimensions and coordinates. (a) Radial line with a fine sire of elliptical cross section (b) Diagram for a thin wire.

The system analyzed in this report is shown in Fig. 1. A transducer in the form of a fine wire with an elliptical cross section is excited with an RF current which generates radial volume waves within the structure. The dc magnetic field is directed along the z axis, and also the fine wire with an elliptical cross section is situated parallel to the z axis. This radial wave propagates perpendicular to the magnetic biasing fields, guided by parallel surfaces of a ferrite disk, and its energy is distributed within the medium. As Ganguly *et al.* pointed out previously, the frequency characteristics of the radiation resistance is influenced considerably by changing the microstrip width. The main subject of this paper is to demonstrate how a change of eccentricity of an elliptical metal cylinder influences the characteristics of the radiation resistance. The radiation pattern of this elliptic cylinder (Ribbon type) excitation possesses some type of directivity. If the eccentricity of the ellipse decreases, the patterns of the radiative power are highly directional, and this directed energy power tends to be confined to the direction of the y-axis shown in Fig. 1(b). It is possible to gain the maximum output power by placing the second thin wire within an area of maximum radiative power.

II. Basic Theory

When the fields are independent of z $(\partial/\partial z = 0)$, for the component e_z of the electric field vector, Maxwell's equation leads to the two-dimensional wave equation

$$\frac{\partial^2 e_z}{\partial x^2} + \frac{\partial^2 e_z}{\partial y^2} + \omega^2 \epsilon \mu_0 \mu_\perp e_z = 0 \tag{1}$$

where $\mu_\perp = \mu - \kappa^2/\mu$, μ and κ are a diagonal and a nondiagonal component of permeability tensor $\tilde{\mu}$, and μ_0 and ϵ are the vacuum permeability and dielectric constant, respectively. Applying the transformation defined by $x = h\cosh(\xi)\cos(\tau)$, $y = h\sinh(\xi)\sin(\tau)$, the wave equation leads to the equation

$$\frac{\partial^2 e_z}{\partial \xi^2} + \frac{\partial^2 e_z}{\partial \tau^2} + 2k^2(\cosh(2\xi) - \cos(2\tau))\, e_z = 0 \tag{2}$$

where $2k = k_1 h$, $k_1^2 = \omega \epsilon^2 \mu_0 \mu$, $h = x_0(1 - \tanh^2(\xi_0))^{1/2}$, and

$$\xi_0 = 0.5 \ln \frac{(x_0 + y_0)}{(x_0 - y_0)}, \text{ (refer to Fig. 1 on } x_0 \text{ and } y_0).$$

The boundary condition is that, at the surface of a metal cylinder

$$e_z = 0, \quad \xi = \xi_0. \tag{3}$$

Here, for physically acceptable reasons, we assume that the fields having periods π, and the lowest mode $m = 0$ is possible to excite, so that (6) has a nonzero value. Defining $q = (k_1 h)^2/4$, the appropriate formal solution of (2) is [3]

$$e_z = A\left(C_{e0}(\xi, q) - \frac{C_{e0}(\xi_0, q)}{M_{e0}^{(2)}(\xi_0, q)} M_{e0}^{(2)}(\xi, q) \right) c_{e0}(\tau, q) \quad (4)$$

where $C_{e0}(\xi, q)$ is modified Mathieu functions of the first kind, $c_{e0}(\tau, q)$ is a periodic Mathieu function of the first kind, and $M_{e0}^{(2)}(\xi, q)$ is a second solution of a Mathieu equation. The function $M_{e0}^{(2)}(\xi, q)$ corresponds to the Hankel function $H_0^{(2)}(z)$, where the $H_0^{(2)}(z)$ is used to represent outgoing waves in problems pertaining to a circular cylinder. From Maxwell's curl equation we have

$$h_\tau = \frac{1}{\iota\omega\mu_0(\mu^2 - \kappa^2) l}\left(-\iota\kappa \frac{\partial e_z}{\partial \tau} + \mu \frac{\partial e_z}{\partial \xi} \right) \quad (5)$$

where $l = h/\sqrt{2}(\cosh(2\xi) - \cos(2\tau))^{1/2}$. The magnetic field h_τ at the surface of the elliptic cylinder is equal to the surface currents j_s. Then, for the total current j, we have

$$j = \oint j_s l \, d\tau = 2\int_0^\pi h\tau(\tau) l \, d\tau. \quad (6)$$

From (6) we have a relation between j and the coefficient A of (4). The radiation resistance R_m is defined by the ratio the radiative power to the square of the current flowing into the metal cylinder. The radiative power flow is given by

$$P_\xi^{sc} = \text{real}\left(-e_z^{sc}\widetilde{h_\tau^{sc}}\right) \quad (7)$$

where, $\widetilde{h_\tau^{sc}}$ is the complex conjugate of the h_τ^{sc}, and

$$e_z^{sc} = -A\frac{C_{e0}(\xi_0, q)}{M_{e0}^{(2)}(\xi_0, q)} M_{e0}^{(2)}(\xi, q) c_{e0}(\tau, q) \quad (8)$$

and $\widetilde{h_\tau^{sc}}$ can be derived from (5) by making use of (8). Thus, the radiation resistance is defined by

$$R_m = \frac{2\int_0^\pi p_\xi^{sc} l \, d\tau}{|j|^2}. \quad (9)$$

From (7), (8), and (9), we have

$$R_m = -\frac{\left(\omega\mu_0(\mu^2 - \kappa^2)\right)^2}{(2\pi\mu)^2} \cdot \frac{1}{\left| \frac{\partial C_{e0}(\xi_0, q)}{\partial \xi} - \frac{C_{e0}(\xi_0, q)}{M_{e0}^{(2)}(\xi_0, q)} \frac{\partial M_{e0}^{(2)}(\xi_0, q)}{\partial \xi} \right|^2} \cdot \frac{1}{\iota\omega_{\mu 0}\mu_\perp} \cdot \left| \frac{C_{e0}(\xi_0, q)}{M_{e0}^{(2)}(\xi_0, q)} \right|^2 \cdot M_{e0}^{(2)}(\xi, q) \cdot \widetilde{\frac{\partial M_{e0}^{(2)}(\xi, q)}{\partial \xi}} \cdot I \quad (10)$$

where

$$I = 2\int_0^\pi |c_{e0}(\tau, q)|^2 \, d\tau. \quad (11)$$

It is to be noted that there is no ξ dependence of R_m in the ξ direction.

In Fig. 2, the radiation resistance R_m is plotted as a function of frequency. In Fig. 2, discontinuities of the curves near the frequency of 2.5 GHz are originated from the use of the approximate formulas for Mathieu functions. For small values of q, we expect the line of the curves in the low frequency regions to be more accurate, while the curves in the high frequency regions should be more reliable for large values of q. Interesting characteristics are shown in Fig. 2 that this transducer near the critical frequency $\omega = \gamma(BH)^{1/2}$. The following trends can be found for the range of parameters considered in this paper: 1) the exciting bandwidth of low frequency side decreases approximately proportional to x_0; 2) the maximum value of R_m decreases with increasing x_0; and 3) a number of zeros of the R_m appear as x_0 increases.

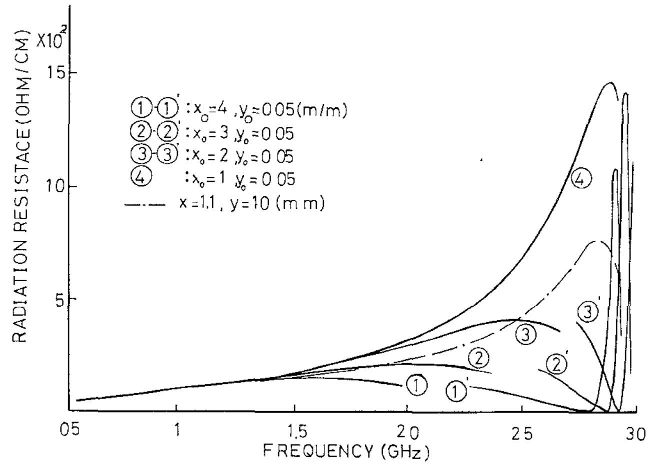


Fig. 2. Radial resistance is plotted as a function of frequency for various dimensions of the thin wire. Other parameters are biasing dc magnetic field $H = 500$ oe, saturation magnetization $4\pi M_s = 1800$ G.

In order to confirm this theory, a curve is plotted in Fig. 2 by a chain line so that this curve shows the resistance of a thin wire with a shape which is almost considered as a circular cylinder ($x_0 = 1.1$ m/m, $y_0 = 1.0$ m/m). This curve agrees quite well with the previous results which were obtained by using Bessel functions. [1]

III. Conclusion

In conclusion, a broad bandwidth ($1 \sim 2$ GHz) radial-wave transducer can be expected to design by making use of a fine wire with elliptical cross section. In view of the above, developments of a suitable radial line should produce extremely wide bandwidth and a magnetically tunable low frequency ($0.5 \sim 1.0$ GHz) microwave transducer.

References

[1] To be published
[2] A K. Ganguly and D. C Webb, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 998–1006, Dec. 1975.
[3] N W McLachlan, *Theory and Application of Mathieu Functions*. New York: Dover, 1964.

A Simple Formula for the Capacitance of a Disc on Dielectric on a Plane

HAROLD A. WHEELER, FELLOW, IEEE

***Abstract*—There is presented a simple explicit formula for the capacitance of a thin circular disc on a dielectric substrate on a plane (so-called "microstrip"). It gives continuous coverage of all shape ratios (r/h) and all**

Manuscript received March 22, 1982; revised June 8, 1982.
The author is with the Hazeltine Corporation, Greenlawn, NY 11740